

# Space Vector Pulse Width Modulation

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**Abstract**— The rapid development of high switching frequency power electronics in the past decade leads towards wider application of voltage source inverters in AC power generation. Therefore, this prompts the need for a modulation technique with less total harmonic distortion (THD) and fewer switching losses. Space vector pulse width modulation (SVPWM) provides a better technique compared to the more commonly used PWM or sinusoidal PWM (SPWM) techniques. SVPWM is a more sophisticated technique for generating a fundamental sine wave that provides a higher voltage, high reduction in the dominant harmonics and lower total harmonic distortion when used in an inverter. In SVPWM the complex reference voltage phasor is processed as a whole, therefore, interaction between three phases is exploited, and this strategy reduces the switching losses by limiting the switching. This paper will analyze the working and design of SVPWM and will provide comparative analysis of improved quality with the conventional methods.

**Index Terms** — space vector pwm, space vector, pulse width modulation.

## 1 INTRODUCTION

PULSE width modulation (PWM) has been studied extensively during the past decades. Many different PWM methods have been developed to achieve the following aims: wide linear modulation range; less switching loss; less total harmonic distortion (THD) in the spectrum of switching waveform; and easy implementation and less computation time. For a long period, carrier-based PWM methods were widely used in most applications. The earliest modulation signals for carrier-based PWM are sinusoidal. The use of an injected zero-sequence signal for a three-phase inverter initiated the research on non-sinusoidal carrier-based PWM. Different zero-sequence signals lead to different non-sinusoidal PWM modulators. Compared with sinusoidal three-phase PWM, non-sinusoidal three-phase PWM can extend the linear modulation range for line-to-line voltages.

With the development of microprocessors, space-vector modulation has become one of the most important PWM methods for three-phase converters. It uses the space-vector concept to compute the duty cycle of the switches. It is simply the digital implementation of PWM modulators. An aptitude for easy digital implementation and wide linear modulation range for output line-to-line voltages are the notable features of space vector modulation. The comprehensive relation of the two PWM methods provides a platform not only to transform from one to another, but also to develop different performance PWM modulators. Therefore, many attempts have been made to unite the two types of PWM methods.

In SVPWM methods, the voltage reference is provided using a revolving reference vector. In this case magnitude and frequency of the fundamental component in the line side are controlled by the magnitude and frequency, respectively, of the reference voltage vector. Space vector modulation utilizes dc bus voltage more efficiently and generates less harmonic dis-

tortion in a three phase voltage source inverter.

## 2 PWM PRINCIPLE

The dc input to the inverter is “chopped” by switching devices in the inverter (bipolar transistors, thyristors, Mosfet, IGBT ...etc). The amplitude and harmonic contents of the ac waveform are controlled by controlling the duty cycle of the switches. This is the basic of the pulse width modulation PWM techniques.

There are several PWM techniques each has its own advantages and also disadvantages. The basic PWM techniques are described briefly in the following subsections. The considered PWM techniques are:

- 1) Sinusoidal PWM (most common)
- 2) Space-Vector PWM

### 2.1 Sinusoidal Pulse width modulation

In this method a triangular (carrier) wave is compared to a sinusoidal wave of the desired fundamental frequency and the relative levels of the two signals are used to determine the pulse widths and control the switching of devices in each phase leg of the inverter. Therefore, the pulse width is a sinusoidal function of the angular position of the reference signal. The basic principle of three phase sinusoidal PWM is shown in Fig. 1. (Refer fig.1)

The sinusoidal PWM is easy to implement using analog integrators and comparators for the generation of the carrier and switching states. However, due to the variation of the sine wave reference values during a PWM period, the relation between reference values and the carrier wave is not fixed.

Depending on whether the signal voltage is larger or smaller than the carrier waveform, either the positive or negative dc bus voltage is applied at the output. Note that over the period of one triangle wave, the average voltage applied to the load is proportional to the amplitude of the signal (assumed constant) during this period.

The resulting chopped square waveform contains a replica of

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the desired waveform in its low frequency components, with the higher frequency components being at frequencies close to the carrier frequency. Notice that the root mean square value of the ac voltage waveform is still equal to the dc bus voltage, and hence the total harmonic distortion is not affected by the PWM process.

The harmonic components are merely shifted into the higher frequency range and are automatically filtered due to inductances in the ac system.

Fig.2 is an example of the SPWM with modulation index more than 1. However, due to the variation of the sine wave reference values during a PWM period, the relation between reference values and the carrier wave is not fixed. This results in existence of harmonics in the output voltage causing undesired low-frequency torque and speed pulsations. The problems associated with SPWM are:

- 1) The machine models and characteristics used are valid only in steady state. This causes the control to allow high peak voltage and current transients. These damage not only the drive dynamic performance but also the power conversion efficiency. Additionally, the power components must be oversized to withstand the transient electrical spikes.
- 2) Great difficulty in controlling the variables with sinusoidal references: PI regulators cannot perform a sinusoidal regulation without damaging the sinusoidal reference, and hysteresis controllers introduce high bandwidth noise into the system that is hard to filter out.
- 3) No three phase system imbalance management. No consideration of the phase interactions.
- 4) Finally, the control structure must be dedicated according to motor type (asynchronous or synchronous).

### 3 SPACE VECTOR PULSE WIDTH MODULATION

Space vector PWM refers to a special switching scheme of the six power semiconductor switches of a three phase power converter. Space vector PWM (SVPWM) has become a popular PWM technique for three-phase voltage-source inverters in applications such as control of induction and permanent magnet synchronous motors. The mentioned drawbacks of the sinusoidal PWM are reduced using this technique. Instead of using a separate modulator for each of the three phases, the complex reference voltage vector  $s$  processed as a whole. Therefore, the interaction between the three motor phases is considered. It has been shown, that SVPWM generates less harmonic distortion in both output voltage and current applied to the phases of an ac motor and provides a more efficient use of the supply voltage in comparison with sinusoidal modulation techniques. SVPWM provides a constant switching frequency and therefore the switching frequency can be adjusted easily. Although SVPWM is more complicated than sinusoidal PWM and hysteresis band current control, it may be implemented easily with modern DSP based control Systems.

### 3.1 Principle of space vector pulse width modulation

Eight possible combinations of on and off patterns may be achieved. The on and off states of the lower switches are the inverted states of the upper ones.

The phase voltages corresponding to the eight combinations of switching patterns can be calculated and then converted into the stator two phase ( $\alpha\beta$ ) reference frames. This transformation results in six non-zero voltage vectors and two zero vectors. The non-zero vectors form the axes of a hexagon containing six sectors (V1 – V6).

The angle between any adjacent two non-zero vectors is 60 electrical degrees. The zero vectors are at the origin and apply a zero voltage vector to the motor. The envelope of the hexagon formed by the non-zero vectors is the locus of the maximum output voltage. SVPWM consists of controlling the stator currents represented by a vector. This control is based on projections which transform a three phase time and speed dependent system into a two co-ordinate (d and q co-ordinates) time invariant system. These projections lead to a structure similar to that of a DC machine control. Field orientated controlled machines need two constants as input references: the torque component (aligned with the q co-ordinate) and the flux component (aligned with d co-ordinate). From Fig. 3

$$V_a = V_m \sin(\omega t)$$

$$V_b = V_m \sin(\omega t - 120)$$

$$V_c = V_m \sin(\omega t + 120)$$

Thus,  $V_s$  can be written as,

$$V_s = V_a + V_b e^{+j\frac{2\pi}{3}} + V_c e^{-j\frac{2\pi}{3}}$$

Solving above equations,

$$V_s = \frac{3}{2} V_m [ \sin\omega t - j \cos\omega t ]$$

Therefore magnitude of  $V_s = \frac{3}{2} V_m$  and it rotates in space by ( $\omega$  rad/sec )

Where  $\omega$  = frequency of three sine waves  $V_a, V_b, V_c$ .

Thus  $V_s = V_x + Vy$ ----from above diagram

In matrix form,

$$\begin{bmatrix} V_x \\ V_y \end{bmatrix} = \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}$$

$$V_x = V_a - \frac{1}{2} [ V_b + V_c ] = \frac{3}{2} V_a \quad \dots\dots\dots 1$$

$$V_y = \frac{\sqrt{3}}{2} [ V_b - V_c ] \quad \dots\dots\dots 2$$

Now consider Fig. 4

$$V_{ao} = V_{an} + V_{no} \quad \dots\dots\dots 3$$

$$V_{bo} = V_{bn} + V_{no} \quad \dots\dots\dots 4$$

$$V_{co} = V_{cn} + V_{no} \quad \dots\dots\dots 5$$

$$\text{Knowing that } V_{an} + V_{bn} + V_{cn} = 0 \quad \dots\dots\dots 6$$

Adding equation 3, 4, 5 we get

$$V_{no} = \frac{1}{3} [ V_{ao} + V_{bo} + V_{co} ] \quad \dots\dots\dots 7$$

Substituting equation 7 in 3, 4 & 5,

$$V_{an} = \frac{2}{3} V_{ao} - \frac{1}{3} [V_{bo} + V_{co}]$$

$$V_{bn} = \frac{2}{3} V_{bo} - \frac{1}{3} [V_{co} + V_{ao}]$$

$$V_{cn} = \frac{2}{3} V_{co} - \frac{1}{3} [V_{ao} + V_{bo}]$$

In Matrix form, hence the maximum output phase voltage and line-to-line voltage that can be achieved by applying SVPWM are:

$$\begin{bmatrix} V_{an} \\ V_{bn} \\ V_{cn} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix} \begin{bmatrix} V_{ao} \\ V_{bo} \\ V_{co} \end{bmatrix} \dots 8$$

### 3.2 Modulation index, time period and calculations of vectors

IJSER Considering Fig. 5 and Fig. 6, we get 8 combinations of switching instances as 2 switches of 3 legs of inverter will give  $2^3=8$ . Among this 6 are active vectors and two are zero vectors as the combination of [1 1 1] and [0 0 0] will give zero vectors.

e.g: (0 0 1)

$$V_{ao} = V_{dc} / 2$$

$$V_{bo} = -V_{dc} / 2$$

Substituting in above matrix,

$$V_{an} = \frac{2}{3} V_{dc}$$

$$\& V_{bn} = V_{cn} = -\frac{1}{3} V_{dc}$$

Therefore,  $V_x = \frac{3}{2} V_{an}$  --- from eq. 1

Thus,  $V_x = \frac{3}{2} \times \frac{2}{3} V_{dc} = V_{dc}$

&  $V_y = \frac{\sqrt{3}}{2} [V_{an} - V_{cn}]$  --- from eq. 2

Therefore  $V_y = 0$

Thus  $V_s = V_{dc} \angle 0$

Similarly for ,

c b a

( 1 1 0 ) which is complimentary of ( 0 0 1 ),

$V_s = V_{dc} \angle 180$

now for,

( 0 1 1 )  $\Rightarrow V_s = V_{dc} \angle 60$

Thus for (1 0 0)  $\Rightarrow V_s = V_{dc} \angle 240$

Now for,

( 0 1 0 )  $\Rightarrow V_s = V_{dc} \angle 120$

Thus for (1 0 1)  $\Rightarrow V_s = V_{dc} \angle 300$

Thus we get a simple relationship between phase & pole voltages.

If  $V_{an}, V_{bn}, V_{cn}$  are sinusoidal, then

$$V_s = M e^{j\omega t}$$

Where  $M \Rightarrow$  modulation index,  $0 < M < 1$

$\omega \Rightarrow$  output frequency

$V_s \Rightarrow$  locus of circle

$V_s$  moves in discrete steps of 60

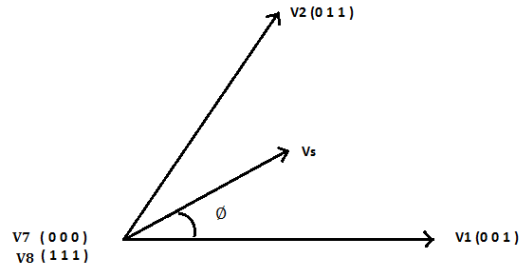


Fig. 7 Space vector in sector 1

Where,  $\phi \Rightarrow$  position of  $V_s$  in x-y plain.

There should be a volt-sec balance which depends on magnitude of  $V_s$ .

$$V_s T_c = V_1 T_1 + V_2 T_2 + V_z T_z$$

( The value of  $V_z T_z$  is always zero )

Where,  $T_c = \frac{T_s}{2} \Rightarrow$  sampling time

If  $T_z = T_c - T_1 - T_2$

This condition is satisfied then it does not matter how long we use ( 0 0 0 ) & ( 1 1 1 )

Maximum value of space vector i.e  $V_{smax} =$  radius of circumscribing circle.

$$= V_{dc} \cos 30$$

$$= \frac{\sqrt{3}}{2} V_{dc} \dots \dots 9$$

Now consider a ratio of fundamental component of SVPWM to square wave..

Let,  $mf = \frac{V1_{sp}}{V1_s} \dots \dots 10$

Where,

$V1_{sp} =$  peak of fundamental of phase voltage of SVPWM

$V1_s =$  peak of fundamental of phase voltage obtained by square wave.

Now  $V_{an}, V_{bn}, V_{cn}$  in terms of  $V_x$  &  $V_y$  are given as,

$V_{an} = \frac{2}{3} V_x$  --- from eq. 1

$V_{bn} = \frac{1}{3} V_x + \frac{1}{\sqrt{3}} V_y$  --- from eq. 2

$V_{cn} = -\frac{1}{3} V_x - \frac{1}{\sqrt{3}} V_y$  --- from eq. 2

Now,

Considering ( 0 0 1 )

$V_{an\ peak} = \frac{2}{3} V_{smax}$

Therefore,  $V_{an\ peak} = \frac{2}{3} \times \frac{\sqrt{3}}{2} V_{dc}$  --- from eq. 9

$V_{an\ peak} = V_{bn\ peak} = V_{cn\ peak} = \frac{V_{dc}}{\sqrt{3}} = 0.577 V_{dc}$

Therefore

$$mf = \frac{0.577 V_{dc}}{\frac{2}{\pi} V_{dc}} = 0.907$$

thus, 90.7% of fundamental component of square wave is available in SVPWM as compared to 78.5% of sine PWM. Sampling time  $T_s$  should be as small as possible. The time period can be shown graphically in Fig. 8.

#### 4 FIGURES

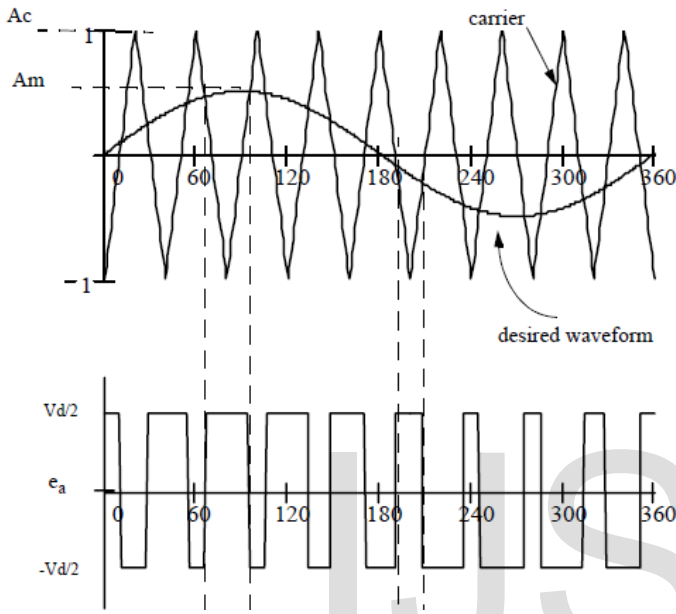


Fig. 1 SPWM.

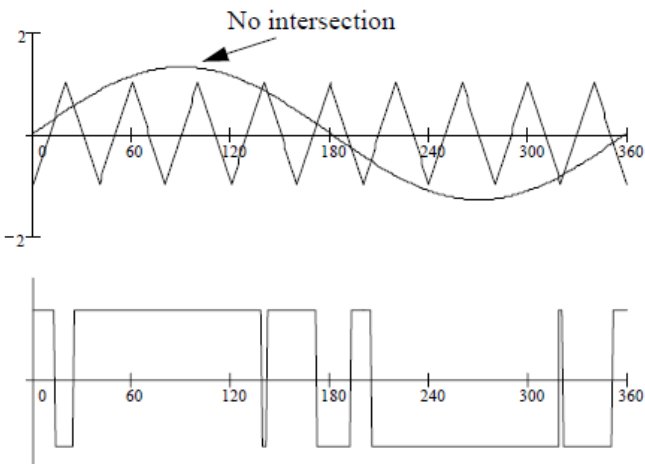


Fig. 2 SPWM with modulation index more than 1.

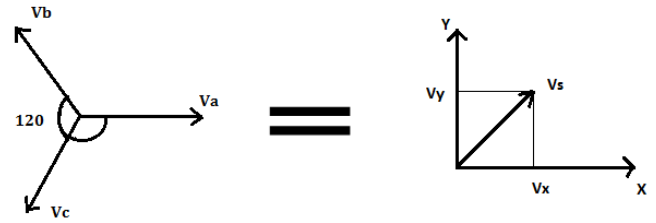


Fig. 3 Space vector

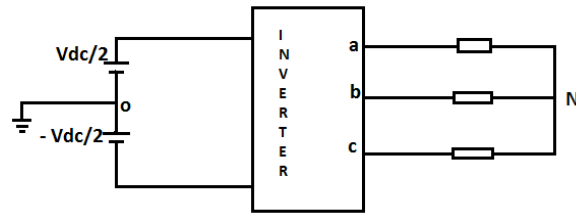


Fig. 4 Simplified inverter circuit for calculation

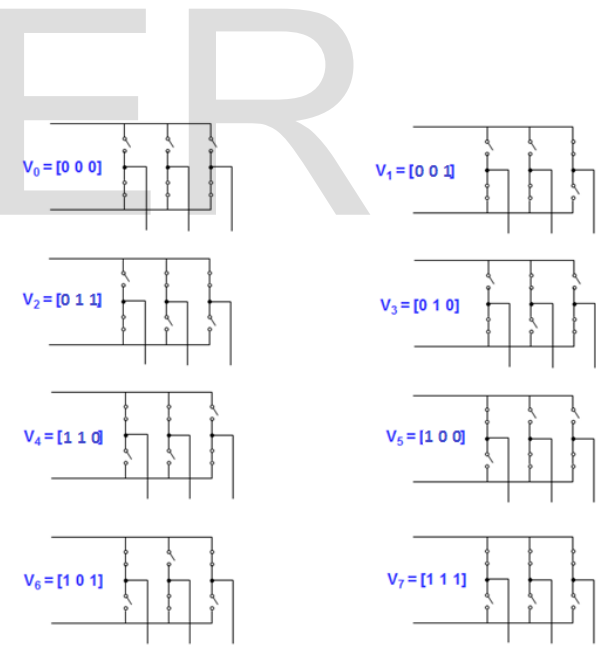


Fig. 5 Switching instances of MOSFET

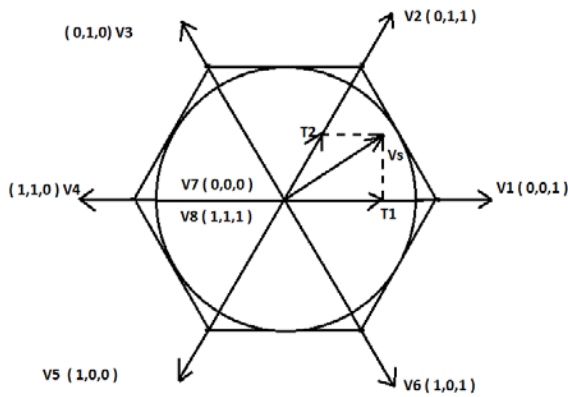


Fig. 6 Phasor representation of Space vector

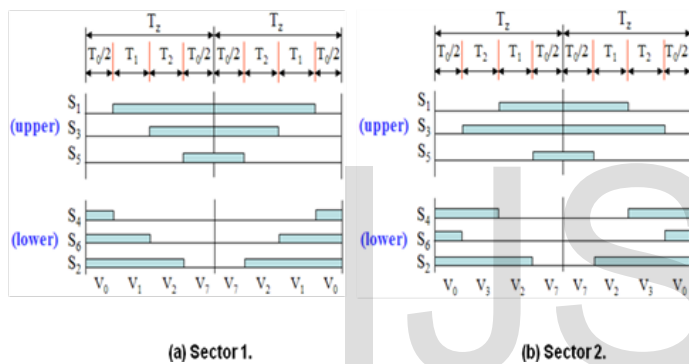


Fig. 8 Space vector switching pattern at Sector 1 and Sector 2

## 5 CONCLUSION

A SVPWM technique based on a reduced computation method was presented. The SVPWM scheme can drive the inverter gating signals from the sampled amplitudes of the reference phase voltages. The switching vectors for the inverter are derived using a simple digital logic which does not involve any complex computations and hence reduces the implementation time. SVPWM drive treats the inverter as a single unit with eight possible switching states, each state can be represented by a state vector in the two-axis space, the eight state vectors formed a hexagon shape with six sectors. The modulation procedure is accomplished by switching the state vectors in each sector by appropriate time intervals which are calculated in a certain sampling time ( $T_s$ ). The linear region in SVPWM is larger than other types of PWM technique, where the modulation index approaches to (90.7%) and the maximum output fundamental is (0.577Vd), whereas, in the SPWM the maximum linear modulation index is (78.54%) and the maximum output fundamental is (0.5Vd). The harmonic analysis of different output voltage and current, in both simulation and experimental results, gives excellent harmonic reduction and harmonic parameters with respect to square-wave inverter. The total losses of low order harmonics can be minimized by

increasing the switching frequency, but in the other hand it may increase the switching losses, therefore, switching frequency must be selected to get minimum total harmonic and switching losses. The SVPWM is a digital modulating technique. Then from the above conclusion and due to simulation and experimental results, the SVPWM can be considered as the best and the optimum of all PWM technique.

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